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Introduction

Our principal goal is to study the dynamics generated by the smooth Hamiltonian function

\[ H = \sum \lambda_i \sin(\phi_i) + \frac{1}{2} \sum \left( \sin(\phi_i) \sin(\phi_j) + \sin(\phi_i) \sin(\phi_j) \right) \]

where \( \phi_i, \lambda_i \) are arbitrary coupling constants satisfying the conditions \( \sin(\phi_i) \neq 0 \neq \sin(\phi_j) \).

The van Diejen systems are multivariate integrable deformations of the translation-invariant Runge-Lenz-Schrödinger (RLS) model. In the present paper, we introduce the Calogero-Moser-Sutherland (CMS) models associated with the BC-type root systems. The eigenfunctions of the hyperbolic n-particle van Diejen model in the open subinterval \( (0,1) \) are given by

\[ Q = \{ (\lambda_1, \lambda_2, \ldots, \lambda_n) \mid \lambda_1 > \lambda_2 > \cdots > \lambda_n > 0 \} \subset \mathbb{R}^n \]

that can be seen as an open Weyl chamber of type BC. The eigenbasis of Q, normalized, and \( \lambda_i \) can be naturally identified with the open subinterval

\[ Q = \{ (\lambda_1, \lambda_2, \ldots, \lambda_n) \mid \lambda_1 > \lambda_2 > \cdots > \lambda_n > 0 \} \subset \mathbb{R}^n \]

Lax representation of the hyperbolic van Diejen system with two coupling parameters


Some relations

Lax representation of the hyperbolic van Diejen system with two coupling parameters

A joint work with B.G. PUSZTAI

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Poisson brackets of the eigenvalues of \( L \)

Theorem. The eigenvalues of the L matrix are in involution.

\[ \left\{ \lambda_1, \lambda_2 \right\} = \sinh(\psi_1) - \sinh(\psi_2) = \sinh(\psi_2) - \sinh(\psi_1) \]

Link to van Diejen's Hamiltonians

The complete set of pairwise commuting functions \( \{ H_i \} \) of van Diejen can be defined by introducing the following two-matrix functions

\[ \phi_i = \frac{1}{2} \ln \left( \frac{Q_i}{P_i} \right) \]

\[ \lambda_i = \frac{1}{2} \ln \left( \frac{P_i}{Q_i} \right) \]

For the coordinates of the model, \( \{ H_i \} \) are expressed through the eigenvalues of our L matrix and van Diejen's commuting Hamiltonians were also demonstrated.

\[ \{ H_1, H_2 \} = \sinh(\psi_1) - \sinh(\psi_2) = \sinh(\psi_2) - \sinh(\psi_1) \]

Physical interpretation of the model

(Particles moving on the half line repelled by each other and the boundary)

Lax representation of the dynamics

Theorem. The Hamiltonian vector field \( X_H \) generated by the van Diejen type Hamiltonian function \( H \) is complete. That is, the maximum number of integrals of each integral case in the whole real axis \( R \).

Temporal asymptotics

Lemma. For an arbitrary smooth solution of the hyperbolic n-particle van Diejen system if the particles move asymptotically far in \( t \rightarrow \infty \). More precisely, for all \( x_t \in \mathbb{R}^n \), we have the asymptotics

\[ \lim_{t \to \infty} x_t = \lim_{t \to \infty} (x_t + (1, 0, 0, \ldots, 0)) = \infty \]

References

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Poisson brackets of the eigenvalues of \( L \)

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Completeness of the Hamiltonian vector field

Let \( X_H \) be defined by

\[ \{ H_1, H_2 \} = \sinh(\psi_1) - \sinh(\psi_2) = \sinh(\psi_2) - \sinh(\psi_1) \]

Theorem. The dynamics of \( L \) is a toy for the dynamics generated by the Hamiltonian \( H \).

Analyzing the dynamics

Observations

For an arbitrary smooth solution of the hyperbolic n-particle van Diejen system if the particles move asymptotically far in \( t \rightarrow \infty \). More precisely, for all \( x_t \in \mathbb{R}^n \), we have the asymptotics

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